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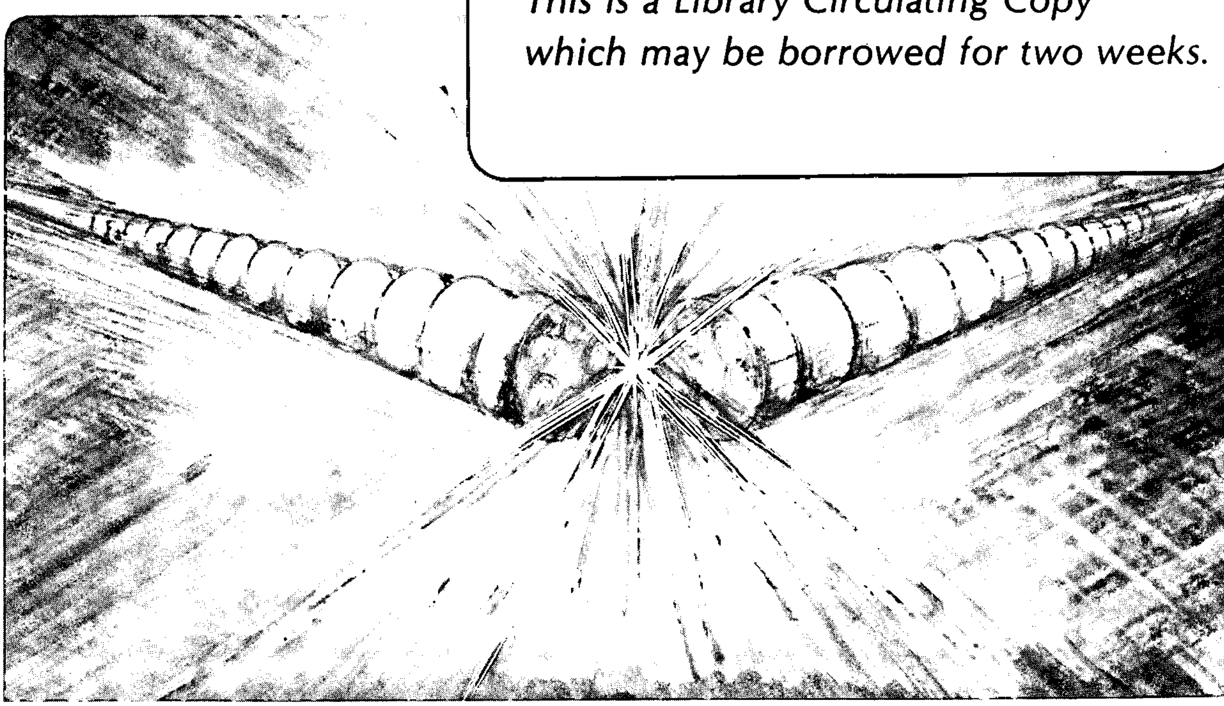

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SCATTERING OF AN ICRF MAGNETOSONIC WAVE BY PLASMA DENSITY TURBULENCE*

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ABSTRACT

A fast ICRF magnetosonic wave, launched into a tokamak plasma, scatters off turbulent density fluctuations in the plasma edge. We use cold-fluid theory to calculate the angular distribution of the scattered wave and find it to be predominantly perpendicular to the incident wavevector for second harmonic majority heating. We calculate the mean free path and find it to be large compared to the size of tokamak devices. Therefore, scattering of ICRF magnetosonic waves by density turbulence is an utterly negligible effect.

INTRODUCTION

The incident magnetosonic wavevector has $k_\parallel \ll k_\perp$ and experimental data indicate that the density turbulence has $k_\parallel \approx 0$. We therefore in our model let $k_\parallel = 0$ for all wavevectors, incident, scattered, and turbulent. The scattering occurs in the plane perpendicular to B_o. Experimental data indicate that the correlation length of the density turbulence is small compared to the magnetosonic wavelength. The density turbulence fluctuates slowly compared to the frequency of the magnetosonic wave, so the turbulent scatterers look stationary to the incident wave, and the scattering is elastic. In the wave field quantities which follow, an $e^{i\omega t}$ time dependence is to be understood. We found many useful ideas which helped us with this calculation in Ishimaru's fine book, Wave Propagation and Scattering in Random Media[1].

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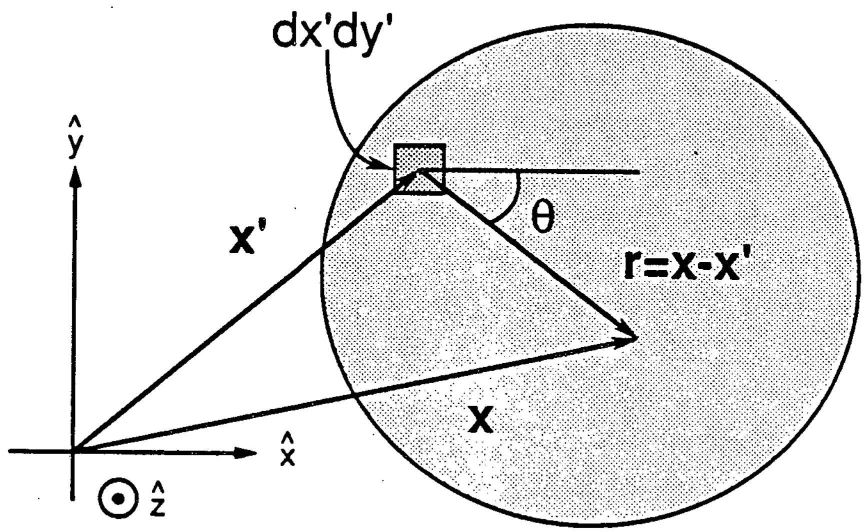
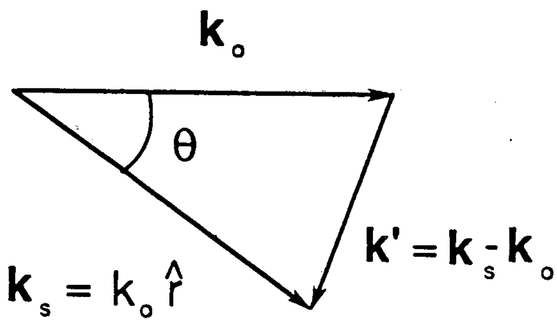
We begin with a few basic symbols and relations:

$$\psi(x,y) \equiv \tilde{B}(x,y)/B_o, \qquad \mu(x,y) \equiv n_t(x,y)/n_o, \qquad \mathbf{k}' \equiv \mathbf{k_{scat}} - \mathbf{k_o},$$
$$\psi(\mathbf{x}) = \psi_{inc}(\mathbf{x}) + \psi_{scat}(\mathbf{x}), \qquad n = n' + \tilde{n}, \qquad \mathbf{k_{scat}} \equiv k_o \hat{r},$$
$$\psi_{inc}(\mathbf{x}) = \psi_o e^{i\mathbf{k_o}\cdot\mathbf{x}}, \qquad n' \equiv n_o + n_t, \qquad {k_o}^2 \equiv \omega^2/{C_A}^2,$$
$${C_A}^2 \equiv {B_o}^2/4\pi M_i n_o, \qquad \Omega_i \equiv eB_o/M_i c, \qquad \bar{\lambda}_o \equiv 1/k_o,$$

$Re\left[e^{-i\omega t}\tilde{B}(x,y)\right]$,	magnetic field of the wave
n_o,	uniform background density
$\tilde{n}$,	density variation due to wave
n_t,	turbulent density fluctuation





From the equations of two–component cold fluid theory, we obtain the following wave equation:

$$(\nabla^2 + k_o^2)\psi(x,y) = \left\{-k_o^2\mu + \left[\nabla\mu - i\frac{\omega}{\Omega_i}(\hat{z}\times\nabla\mu)\right]\cdot\nabla\right\}\psi(x,y)$$

We make the Rayleigh–Born Approximation on the wavefunction in the above equation by letting $\psi_{scat} \ll \psi_o$. Next we perform a far field approximation on the Green's Function to obtain the following scattered wavefunction:

$$\psi_s(\mathbf{x}) = \sqrt{\frac{k_o}{8\pi}}\left(i\hat{k}_o - \frac{\omega}{\Omega_i}\hat{z}\times\hat{k}_o\right)\cdot\nabla_{\mathbf{x}}\int d^2x'\ \mu(\mathbf{x}')\ \frac{e^{ik_o[\hat{k}_o\cdot\mathbf{x}'+|\mathbf{x}-\mathbf{x}'|]}}{|\mathbf{x}-\mathbf{x}'|^{\frac{1}{2}}}$$

The spectral density $S(\mathbf{k}';\mathbf{x}')$ is the Wigner Function of the relative density turbulence, $\mu(\mathbf{x}')$. It represents the *frequency- integrated wavevector content* of the relative density turbulence at position $\mathbf{x}'$. The $\langle\quad\rangle$ indicates a time average over a time which is long compared to the timescale of the density turbulence.

$$S(\mathbf{k}';\mathbf{x}') \equiv \int d^2s\ e^{-i\mathbf{k}'\cdot\mathbf{s}}\left\langle\mu(\mathbf{x}'+\frac{1}{2}\mathbf{s})\ \mu(\mathbf{x}'-\frac{1}{2}\mathbf{s})\right\rangle$$

We construct the Wigner Function of the scattered wave function and use it to obtain the following expression for the scattered wave intensity:

$$\langle|\psi_s(\mathbf{x})|^2\rangle = \frac{k_o^3}{8\pi}\int\frac{d^2x'}{r}\ g(\theta)\ S(\mathbf{k}';\mathbf{x}')\ |\psi_o|^2, \qquad g(\theta) \equiv cos^2\theta + (\frac{\omega}{\Omega_i})^2 sin^2\theta$$

The strength of the density turbulence, $\langle\mu^2(\mathbf{x}')\rangle$ is related to the spectral density and to the wavenumber spread k_n of the density turbulence as follows:

$$\langle\mu^2(\mathbf{x}')\rangle = \int\frac{d^2k'}{(2\pi)^2}\ S(\mathbf{k}';\mathbf{x}') \equiv \frac{k_n^2}{(2\pi)^2}S(\mathbf{k}'\rightarrow 0;\mathbf{x}'),$$

$$k_n^2 \equiv \int d^2k'\ \frac{S(\mathbf{k}';\mathbf{x}')}{S(\mathbf{k}'\rightarrow 0;\mathbf{x}')}$$

We obtain the scattering cross section density from the scattered wave intensity and use it to construct the mean free path,

$$\frac{1}{l(\mathbf{x}')} \equiv \int d\theta\, \sigma(\mathbf{x}';\theta),$$

which may be expressed in terms of the wavenumber spread k_n of the density turbulence as follows:

$$\frac{\lambda\!\!\!^{-}_o}{l} = \frac{\pi^2}{2}\left[1 + (\frac{\omega}{\Omega_i})^2\right] \langle \mu^2 \rangle \left(\frac{k_o}{k_n}\right)^2$$

Using Ritz *et al.*[2] data from TEXT we obtain the following:

$k_o \approx 0.1\ cm^{-1}$,	at the plasma edge
$k_n \approx 6\ cm^{-1}$,	spread in turbulent wavenumbers
$\mu \approx 0.2$,	turbulence strength at the plasma edge
$\omega/\Omega_i \doteq 2$,	second harmonic heating
$\lambda\!\!\!^{-}_o/l \approx 10^{-4}$,	from above equation
$l \approx$ **1 km**,	**mean free path**

CONCLUSION

Two dimensional cold–fluid theory predicts a mean free path large compared to the size of tokamak devices. Scattering of incident ICRF magnetosonic waves by turbulent density fluctuations is **utterly negligible.**

REFERENCES

1. A. Ishimaru, Wave Propagation and Scattering in Random Media (Academic Press, N.Y., 1978), p. 329.
2. C.P. Ritz *et al.*, Nucl. Fusion 27, 1125 (1987).

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